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ON A PARTICULAR LIFE TESTING PROBLEM

BY

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On a Particular Life Testing Problem

A. John Petkau[*]

1. INTRODUCTION

Consider the usual life testing situation in which observations become available in an ordered manner. In this situation, it is natural to consider the possibility of terminating the life test at an early stage; say, after the first r out of the total of n observations have been recorded. Any such procedure has the advantage that it may lead to a decision in a shorter period of time and with fewer observations than a procedure which involves observing what happens to all the items being tested.

The special case in which the underlying lifetime distribution is assumed to be exponential with unknown location parameter β and unknown scale parameter σ has recently been considered by Perng (1977). He argues that a problem which will sometimes arise in practice is that of testing the simple null hypothesis $H_0 : \theta \in \Omega_0$ versus the composite alternative $H_1 : \theta \in \Omega - \Omega_0$ where $\theta = (\beta, \sigma)$, $\Omega = \{\theta \mid 0 \leq \sigma \leq \sigma_0, \ \beta \geq \beta_0\}$ and $\Omega_0 = \{\theta \mid \sigma = \sigma_0, \ \beta = \beta_0\}$. Without loss of generality, we will assume from this point on that $\beta_0 = 0$ and $\sigma_0 = 1$.

Perng argues (Perng, 1977, p.1401) that 'the likelihood ratio method does not give us any "neat" test for this problem, since the likelihood ratio test statistic is rather complicated, and the distribution of the likelihood ratio test statistic under H_0 is not known' and consequently is motivated to look for other approaches to the problem. He proposes a test procedure based on Fisher's method of combining independent test statistics (see, for example, Fisher 1950, pp. 99-101)

and proves that the proposed procedure is asymptotically optimal in the sense of Bahadur efficiency.

In Section 2 of this paper Perng's test procedure is described in detail. Critical values for this test are available and the test appears to be straightforward to carry out. Unfortunately, as is pointed out in Section 2, difficulties arise in attempting to carry out the test in practice. An alternate test procedure is proposed which is truly straightforward to carry out. Petkau (1978) has already shown that this alternate test is asymptotically suboptimal in the sense of Bahadur efficiency. Since neither of these alternate test procedures is entirely satisfactory, we return to consideration of the likelihood ratio method. The likelihood ratio test statistic is determined and the appropriate critical values for carring out the test are provided. In addition, the asymptotic null distribution of this test statistic is determined. The critical regions and the power functions of these three procedures are briefly compared in Section 3.

2. TEST PROCEDURES

Suppose $X_1, X_2, \cdots, X_n$ are the order statistics of a random sample of size n from the exponential distribution with probability density function

$$f_\theta(z) = \frac{1}{\sigma} e^{-(z-\beta)/\sigma} \qquad \text{for } z \geq \beta ,$$

where $\theta = (\beta, \sigma)$. We consider test procedures based on the first $r(1 < r \leq n)$ order statistics for testing

$$H_0 : \theta \varepsilon \Omega_0 \quad \text{versus} \quad H_1 : \theta \varepsilon \Omega - \Omega_0 ,$$

where $\Omega = \{(\beta, \sigma) \mid \beta \geq 0, 0 < \sigma \leq 1\}$ and $\Omega_0 = \{(0,1)\}$.

Define the statistics V_n and U_n as follows

$$V_n = 2[\sum_{i=1}^{r} (X_i - X_r) + (n-r)(X_r - X_1)] ,$$

$$U_n = 2nX_1 .$$

$$(2.1)$$

In the general case, V_n/σ and $2n(X_1-\beta)/\sigma$ are independently distributed as $\chi^2_{(2r-2)}$ and $\chi^2_{(2)}$ respectively and, thus, when H_0 is true, the same is true of V_n and U_n. Small observed values of V_n and large observed values of U_n constitute evidence against H_0.

Perng (1977, p. 1402) defines the statistic

$$Q_n = -2 \log P_{H_0}(U_n \geq u) - 2 \log P_{H_0}(T_n \geq t) \qquad (2.2)$$

where $T_n = V_n^{-1}$ and u, t are the observed values of U_n and T_n

respectively and proposes the test procedure

$$\text{Reject } H_0 \text{ if } Q_n \geq c_Q \ . \tag{2.3}$$

Since Q_n is distributed as $\chi^2_{(4)}$ under H_0 , the critical value $c_Q = \chi^2_{4,1-\alpha}$ can be determined from a chi-square table. However, in order to carry out this test procedure, one must evaluate both of the probabilities appearing in the definition of the statistic Q_n . $P_{H_0}(U_n \geq u)$ can be easily determined since, under H_0 , $U_n/2$ is exponentially distributed with mean one. On the other hand, since chi-square tables contain only a few, and typically very few, selected precentiles, the probability $P_{H_0}(T_n \geq t)$ cannot, in general, be accurately determined. This difficulty is familiar to even the most casual user of statistical methods who may at some time have attempted to determine an observed P-value from a chi-square table. Approximations, such as the Wilson-Hilferty approximation, might serve to alleviate this difficulty some-what (for interesting related work, see Hoaglin (1977)), however, it is clear that while Perng's test is easy to carry out in theory, difficulties arise in attempting to carry out the test in practice.

Since V_n and U_n (see (2.1)) are, under H_0 , independently distributed as $\chi^2_{(2r-2)}$ and $\chi^2_{(2)}$ respectively, and since small values of V_n and large values of U_n are evidence against H_0 , an alternate test which immediately suggests itself is to define the statistic

$$F_n = (r-1)U_n/V_n \tag{2.4}$$

and to use the procedure

$$\text{Reject } H_0 \text{ if } F_n \geq c_F \, . \qquad\qquad (2.5)$$

Under H_0 , F_n has a F distribution with 2 and $2r - 2$ degrees of freedom and the critical value $c_F = F_{2,2r-2,1-\alpha}$ can be determined from a table of the F distribution. This alternate test is truly straight forward to carry out. Unfortunately, at least for large sample sizes this test will perform poorly in comparison to Perng's test since, as has been shown by Petkau (1978), this alternate test is asymptotically suboptimal.

Since neither of the test procedures considered above is entirely satisfactory, we now consider the likelihood ratio test procedure. In what follows, X_r^* denotes the vector of the first r order statistics. The marginal joint density of the first r order statistics (Epstein and Tsao, 1953) is given by

$$g_\theta(x_r^*) = \frac{n!}{(n-r)! \, \sigma^r} \exp\{-[\sum_{i=1}^{r} (x_i-\beta) + (n-r)(x_r-\beta)]/\sigma\} \qquad \text{for } x_1 \geq \beta \, ,$$

where $\theta = (\beta,\sigma)$. Defining

$$\lambda_n(x_r^*) = \frac{\displaystyle\sup_{\theta \in \Omega_0} g_\theta(x_r^*)}{\displaystyle\sup_{\theta \in \Omega} g_\theta(x_r^*)} \, ,$$

it is easy to verify that this likelihood ratio statistic can be written (see(2.1)) as

$$\lambda_n(X_r^*) = \exp(-U_n/2) \qquad\qquad\qquad \text{for } V_n > 2r \text{ ,}$$
$$= \exp(-U_n/2)\{(V_n/2r)\exp[1-(V_n/2r)]\}^r \qquad \text{for } V_n \leq 2r \text{ .} \tag{2.6}$$

The usual likelihood ratio test procedure is

$$\text{Reject } H_0 \text{ if } \lambda_n \leq k_L \text{ .} \tag{2.7}$$

Perhaps a more convenient form is given by

$$-2 \log \lambda_n(X_r^*) = U_n \qquad\qquad\qquad\qquad \text{for } V_n > 2r \text{ ,}$$
$$= U_n - 2r\{1 - (V_n/2r) + \log(V_n/2r)\} \qquad \text{for } V_n \leq 2r \text{ ,} \tag{2.8}$$

with the corresponding procedure

$$\text{Reject } H_0 \text{ if } -2 \log \lambda_n \geq -2 \log k_L = c_L \text{ .} \tag{2.9}$$

It should be noted that this test statistic is easy to evaluate and it reamins only to evaluate the critical value k_L , or equivalently, c_L . In what follows $\psi_m(x)$, $\Psi_m(x)$ denote the density function and distribution function respectively of a random variable having a gamma distribution with parameters m and 1 , that is,

$$\psi_m(x) = \frac{1}{\Gamma(m)} x^{m-1} e^{-x} \qquad\qquad \text{for } x > 0 \text{ ,}$$

$$\Psi_m(x) = \int_0^x \psi_m(y)\,dy \text{ .}$$

We also recall that if Z is distributed as $\chi^2_{(2m)}$, then $Z/2$ is distributed as a gamma with parameters m and 1 . The following theorem evaluates the critical value k_L appearing in (2.7).

Lemma 2.1. The appropriate critical value k_L for a size α $(0<\alpha<1)$ test is given by

$$k_L = [x_0 e^{1-x_0}]^r \qquad (2.10)$$

where x_0 is the unique solution between 0 and 1 of

$$[xe^{1-x}]^r \{1-\Psi_{r-1}(r)+(1/x-1)r\psi_{r-1}(r)\} + \Psi_{r-1}(rx) = \alpha . \qquad (2.11)$$

Proof: For $0 < t < 1$, we have

$$P_{H_0}(\lambda_n(X_r^*) \le t) = P_{H_0}(-\log \lambda_n(X_r^*) \ge -\log t) ,$$

$$= \int_{-\infty}^{\infty} P_{H_0}(-\log \lambda_n(X_r^*) \ge -\log t \mid V_n/2r = x) dP_{H_0}(V_n/2r \le x) ,$$

which, by (2.11) and the independence of U_n and V_n under H_0 , can be written

$$= \int_0^1 P_{H_0}(U_n/2 \ge -\log t + r[1 - x + \log x]) dP_{H_0}(V_n/2r \le x)$$

$$+ \int_1^{\infty} P_{H_0}(U_n/2 \ge -\log t) dP_{H_0}(V_n/2r \le x) .$$

For fixed t, $0 < t < 1$, examine $h(x) = -\log t + r[1-x+\log x]$.

Since $h(0) = -\infty$, $h(1) = -\log t > 0$ and $h(x)$ is monotone increasing

for x between 0 and 1, there is a unique solution, x_0, between

0 and 1 of the equation $h(x) = 0$ which can be written as

$$t = [xe^{1-x}]^r . \qquad (2.12)$$

Then

$$P_{H_0}(\lambda_n(X_r^*) \leq t) = \int_0^{x_0} dP_{H_0}(V_n/2r \leq x) + \int_{x_0}^1 P_{H_0}(U_n/2 \geq -\log t + r[1-x+\log x]) dP_{H_0}(V_n/2r \leq x)$$

$$+ P_{H_0}(U_n/2 \geq -\log t) \int_1^\infty dP_{H_0}(V_n/2r \leq x) ,$$

$$= \Psi_{r-1}(rx_0) + t\, re^{-r} \int_{x_0}^1 x^{-r} e^{rx} \psi_{r-1}(rx) dx$$

$$+ t[1 - \Psi_{r-1}(r)] ,$$

which can be written as

$$P_{H_0}(\lambda_n(X_r^*) \leq t) = \Psi_{r-1}(rx_0) + t\{1-\Psi_{r-1}(r) + r(1/x_0-1)\psi_{r-1}(r)\}. \qquad (2.13)$$

This gives the null distribution function of the likelihood ratio statistic
which leads directly to the desired result.

It should be noted that the critical values k_L, or equivalently
$c_L = -2 \log k_L$, are functions of α and r and do not depend upon n.

Although equations (2.10) - (2.11) appear rather complicated, the availability of standard numerical routines which evaluate the incomplete gamma integral permits the easy evaluation of the critical values by means of an iterative procedure. Indeed, if desired, the null distribution of the likelihood ratio statistic could be examined in detail. Critical values c_L for selected values of α and r are provided in Table 1.

Table 1 about here

It is of interest to examine the asymptotic $(r \to \infty)$ null distribution of minus two times the log of the likelihood ratio test statistic. A cursory glance at Table 1 reveals that convergence to this limiting distribution, whatever it be, is quite slow. Naively one might expect this limiting distribution to be chi-square, as is customary. That this is not quite the case is established in the following theorem.

<u>Theorem 2.1</u>. The asymptotic $(r \to \infty)$ null distribution of $-2 \log \lambda_n(X_r^*)$ is a mixture of $\chi^2_{(2)}$ and $\chi^2_{(3)}$, each with probability $1/2$.

Proof: Replacing t by $e^{-t/2}$ in (2.12) and (2.13), we have

$$P_{H_0}(-2 \log \lambda_n(X_r^*) \geq t) = \Psi_{r-1}(rx_0) + e^{-t/2}\{1 - \psi_{r-1}(r) + r(1/x_0 - 1)\psi_{r-1}(r)\} \quad (2.14)$$

for $t > 0$, where x_0 is the unique solution between 0 and 1 of the equation

$$e^{-t/2} = [x e^{1-x}]^r . \quad (2.15)$$

In order to evaluate the limit as $r \to \infty$ in (2.14), we examine the behavior of x_0 as $r \to \infty$. Writing (2.15) in the form

$$1 - x + \log x = -t/2r \, ,$$

it is clear that as $r \to \infty$, $x_0 \to 1$. Simple expansions yield that as $r \to \infty$,

$$x_0 \sim 1 - \Delta + \Delta^2/3 + \Delta^3/12 + \cdots \, , \qquad (2.16)$$

where $\Delta = (t/r)^{1/2}$. In the following W_r denotes a gamma random variable with parameters $r - 1$ and 1 and $Z_r = [W_r - (r-1)]/(r-1)^{1/2}$. It is well known that Z_r has an asymptotic distribution which is standard normal. Examining the terms in (2.14), we have as $r \to \infty$,

$$\Psi_{r-1}(r) = P(W_r \le r) = P(Z_r \le (r-1)^{-1/2}) \to \Phi(0) = 1/2 \, ,$$

$$\Psi_{r-1}(rx_0) = P(W_r \le rx_0) = P(Z_r \le [r(x_0-1) + 1]/(r-1)^{1/2}) \to \Phi(-t^{1/2}) \, ,$$

where in the latter we have made use of (2.16). Further, use of (2.16) together with Stirling's formula yields that as $r \to \infty$,

$$r(1/x_0-1)\psi_{r-1}(r) \longrightarrow t^{1/2}/(2\pi)^{1/2} \, .$$

Collecting these results, we obtain

$$\lim_{r\to\infty} P_{H_0}(-2 \log \lambda_n(X_r^*) \ge t) = \Phi(-t^{1/2}) + \frac{1}{2} e^{-t/2} + \frac{t^{1/2}e^{-t/2}}{\sqrt{2\pi}} \, .$$

Differentiating this expression yields the asymptotic null density as

$$\frac{1}{2} \left\{ \frac{1}{2} e^{-t/2} + \frac{t^{1/2} e^{-t/2}}{\sqrt{2\pi}} \right\} ,$$

which is the desired result.

It is easy to verify that the $(1-\alpha)$th quantile of this asymptotic distribution is given by c , say, where $c/2$ is the solution of the equation

$$\Psi_{3/2}(x) - e^{-x} = 1 - 2\alpha . \tag{2.17}$$

These critical values are displayed for selected values of α in the last line of Table 1. It should be noted that since the convergence of the critical values c_L to the asymptotic critical values c is quite slow, this asymptotic distribution is not particularly useful for practical purposes. Indeed the magnitude of the effect of using the asymptotic critical values as approximations to the actual critical values can be roughly estimated for any fixed r from Table 1. If desired, this effect could be examined in detail by evaluating (2.12) - (2.13) with $\exp(-c/2)$ replacing t .

The availability of critical values for the likelihood ratio test makes this procedure easy to carry out. None of the practical difficulties which arise in carrying out Perng's test are encountered. From this point of view, both the F test and the likelihood ratio test are preferable to Perng's test. On the other hand, Perng's test is known to be asymptotically optimal in the sense of Bahadur efficiency while the F test is asymptotically suboptimal. Further, Bahadur and

Raghavachari (1972) have shown that any test procedure based on a likelihood

ratio test statistic is, under regularity conditions, asymptotically

optimal. Although these asymptotic considerations rule out the F test

as a viable competitor to either Perng's test or the likelihood ratio

test for all large enough sample sizes, no preference is indicated between

these latter two tests. Moreover, the implication of these asymptotic

considerations for the fixed sample size case is unclear. In the next

section we consider the fixed sample size properties of these three test

procedures.

3. COMPARISON OF TEST PROCEDURES

Since critical values for the likelihood ratio test are now available, this procedure would seem to be preferred to Perng's test in view of the practical difficulties encountered in carrying out the latter. On the other hand, since Perng's test is asymptotically optimal, it is of interest to compare the behavior of Perng's test with that of the likelihood ratio test in fixed sample sizes. Further, it is of interest to determine whether the asymptotic inferiority of the F test is also evident in the fixed sample size case. In this section we examine and compare the critical regions and the power functions of the three test procedures.

Recalling (2.1), we now define

$$Y_n = U_n/2 ,$$
$$Z_n = V_n/2r .$$

From the distribution theory of Section 2, we have

$$Y_n \overset{d}{=} n\beta + \sigma \cdot W$$

$$Z_n \overset{d}{=} \frac{\sigma}{r} \cdot W_{r-1} \tag{3.1}$$

where W and W_{r-1} are independent random variables having (unit) exponential and gamma (with parameters $r - 1$ and 1) distributions respectively. While the distribution of Y_n is the same for all n (except for the shifting of the origin to the point $n\beta$), the distribution of Z_n becomes degenerate at σ as $r \to \infty$. We will now describe

the critical regions of the test procedures of Section 2 in the (Y_n, Z_n) plane. Note that Y_n and Z_n are both positive random variables.

From (2.8)-(2.9), the critical region for the likelihood ratio test is given by the union of the two sets A and B defined by

$$A = \{z>1\} \cap \{y \geq c_L/2\} ,$$

$$B = \{z \leq 1\} \cap \{y - r[1-z + \log z] \geq c_L/2\} .$$

Since $1 - z + \log z \leq 0$ for $z \leq 1$, the set B contains all (y,z) pairs for which $z \leq 1$ and $y \geq c_L/2$. Thus, all pairs for which $y \geq c_L/2$ and no pairs for which both $y < c_L/2$ and $z > 1$ are contained in the critical region. For the remaining pairs, namely those with $z \leq 1$ and $y < c_L/2$, only those points below (or on) the curve $z = z(y)$ defined by

$$y - r[1 - z + \log z] = c_L/2 \tag{3.2}$$

are in the critical region. Examining this curve, we have $z(c_L/2) = 1$ and $z(0) = z_L(c_L, r)$, say, is the unique number between 0 and 1 which is the solution of

$$r[1 - z + \log z] + c_L/2 = 0 .$$

Further, $\frac{dz(y)}{dy} = z/[r(1-z)]$ which is positive for $z < 1$ and approaches ∞ as $z \to 1$, that is, as $y \to c_L/2$. Thus the curve $z(y)$ is monotone increasing for $0 \leq y < c_L/2$ and fits smoothly with the vertical line

$y = c_L/2$ at the point $(c_L/2, 1)$. Indeed, a simple expansion yields that for $z(y)$ defined by (3.2),

$$z(y) \sim 1 - (2\epsilon/r)^{1/2}$$

when $y = c_L/2 - \epsilon$, where ϵ is a small positive quantity.

We note that, for fixed r, $z_L(c_L,r)$ decreases as c_L increases, that is, as the size of the test decreases. Further, for fixed α, $c_L \to c$ defined by (2.17) as $r \to \infty$. Thus, $c_L/r \to 0$ as $r \to \infty$ and consequently $z_L(c_L,r) \to 1$ as $r \to \infty$. As r increases, the critical region of the likelihood ratio test approaches the set of all points in the first quadrant not satisfying both $y \leq c/2$ and $z \geq 1$.

Remark: Given this simple limiting form of the critical region, at first glance it may seem attractive to use this limiting region as an approximation to the critical region for fixed r. That this should not be done is clear upon evaluation of the size of the resulting test

$$P_{H_0}(\text{reject}) = 1 - P_{H_0}(Y_n \leq c/2) \cdot P_{H_0}(Z_n \geq 1)$$
$$= 1 - (1 - e^{-c/2}) \cdot (1 - \Psi_{r-1}(r)) .$$

Now $\Psi_{r-1}(r)$ is monotone decreasing in r (for $r \geq 2$) and $\Psi_{r-1}(r) \to 1/2$ as $r \to \infty$. Thus, for the test which corresponds to this critical region we have

$$1 - (1-e^{-c/2})(1-e^{-2}) \geq P_{H_0}(\text{reject}) \geq 1 - (1-e^{-c/2})/2 \geq 1/2 .$$

Turning to consideration of Perng's test, we see from (2.2) - (2.3) that the critical region is the set of all (y,z) pairs satisfying

$$y - \log \Psi_{r-1}(r \cdot z) \geq c_Q/2 \ . \tag{3.3}$$

Since $- \log \Psi_{r-1}(r \cdot z) > 0$ for $z > 0$, all (y, z) pairs for which $y \geq c_Q/2$ are contained in the critical region. Examining the curve $z = z(y)$ defined by equality in (3.3), we have $z(c_Q/2) = \infty$ and $z(0) = z_Q(c_Q, r)$, say, is the unique positive number satisfying

$$- \log \Psi_{r-1}(r \cdot z) = c_Q/2 \ .$$

Recall that $c_Q = \chi^2_{r, 1-\alpha}$. Since $\Psi_{r-1}(r)$ is monotone decreasing and approaches $1/2$ as $r \to \infty$, $z_Q(c_Q, r) < 1$ provided that $c_Q/2 - \log 2 > 0$ which will certainly be the case for any α of interest. Further, $\dfrac{dz(y)}{dy} = \dfrac{\Psi_{r-1}(rz)}{r\psi_{r-1}(rz)}$ which is positive and approaches ∞ as $z \to \infty$, that is, as $y \to c_Q/2$. Thus, the critical region consists of those points below (or on) the curve $z(y)$ which is monotone increasing and increases without bound as $y \to c_Q/2$.

We note that, for fixed r, $z_Q(c_Q, r)$ decreases as c_Q increases, that is, as the size of the test decreases. Further, it is clear that, for fixed α, $z_Q(c_Q, r) \to 1$ as $r \to \infty$. For purposes of comparing the critical region of Perng's test to that of the likelihood ratio test, it is of interest to determine the point $(y^*, 1)$ which is on the curve defined by (3.3). We have

$$y^* = y^*(c_Q, r) = \log \Psi_{r-1}(r) + c_Q/2 \ .$$

We assume as before that $c_Q/2 - \log 2 > 0$ which insures that the right hand side is positive. As r increases, $\Psi_{r-1}(r) \to 1/2$ and we have

$y^*(c_Q, r) \to c_Q/2 - \log 2$. As r increases, the critical region of Perng's test approaches the set of all points in the first quadrant not satisfying both $y \leq c_Q/2 - \log 2$ and $z \geq 1$.

The critical region of the F test is, from (2.4) - (2.5), the set of all (y, z) pairs satisfying

$$y \geq rc_F z/(r-1) , \qquad (3.4)$$

where $c_F = F_{2,2r-2,1-\alpha}$. The critical region consists of those points below (or on) the straight line $z(y)$ defined by equality in (3.4). Note that, as $r \to \infty$, $rc_F/(r-1)$ decreases monotonically to $\chi^2_{2,1-\alpha}/2 = -\log \alpha$.

The above considerations indicate that the critical regions corresponding to Perng's test and the likelihood ratio test are, for fixed r and α, qualitatively quite similar. This suggests that the power functions of these two test procedures are quite similar. The F test, on the other hand, is defined by quite a different type of critical region which suggests the possibility of a very different power function. The boundaries of the critical regions for these three test procedures can quite easily be evaluated for any fixed r and α although equations (3.2) and (3.3) must be solved in an iterative manner. These critical regions are displayed for the case $\alpha = 0.05$ for selected values of r in Figure 1.

Figure 1 about here

Although the form of the critical regions of the test procedures gives us some insight into the nature of the small sample properties of

these procedures, it would clearly be of interest to obtain explicit
expressions for the power functions of these procedures. These power
functions will in each case depend upon the number of observations, n ,
and the number of order statistics being observed, r , in addition to
the parameter pair (β, σ) and the size of the test. The straightforward
calculations leading to these power functions are not displayed; the
results are merely summarized in the following lemma.

Lemma 3.1. (a) Likelihood ratio test:

Set $\Delta = (c_L/2 - n\beta)/\sigma$. The power function, P_{LR} ,
is given by

$$P_{LR} = 1 \qquad \qquad \text{for } \Delta \leq 0$$

$$= e^{-\Delta}[1 - \Psi_{r-1}(r/\sigma)] + \Psi_{r-1}(rh/\sigma) + \tag{3.5}$$

$$\frac{(r-1)}{[r(1-\sigma)/\sigma+1]} e^{-\Delta}\psi_r(r/\sigma)[h^{-[r(1-\sigma)/\sigma+1]} - 1]$$

$$\text{for } \Delta > 0 ,$$

where h is the unique solution between 0 and 1 of

$$r[1 - h + \log h] = -\sigma \Delta . \tag{3.6}$$

(b) Preng's test:

Set $\gamma = (c_Q/2 - n\beta)/\sigma$. The power function, P_{PR} ,
is given by

$$P_{PR} = 1 \qquad\qquad\qquad\qquad\qquad\text{for } \gamma \leq 0 ,$$

$$\text{(3.7)}$$

$$= \Psi_{r-1}(v/\sigma) + \sigma^{-1}e^{-\gamma} \int_v^\infty \frac{\psi_{r-1}(w/\sigma)}{[\Psi_{r-1}(w)]^{1/\sigma}} \; dw \qquad \text{for } \gamma > 0 ,$$

where v is the unique positive solution of

$$\log \Psi_{r-1}(v) = -\sigma\gamma . \qquad\qquad\qquad\qquad \text{(3.8)}$$

(c) F test:

Set $\xi = \alpha^{1/(r-1)}$ and $\eta = n\beta/\sigma$. The power function, P_F , is given by

$$P_F = \Psi_{r-1}[\eta\xi/(1-\xi)] + \alpha e^{\eta}\{1 - \Psi_{r-1}[\eta/(1-\xi)]\} . \qquad \text{(3.9)}$$

Remark: In evaluating P_F we have made use of the fact that $[1 + F_{2,2r-2,1-\alpha}/(r-1)]^{-(r-1)} = \alpha$.

It should be noted that each of these power functions depends upon n and β only through $n\beta$ as was clear, of course, from (3.1). Further, the power function of the F test depends upon the parameters n , β and σ only through $\eta = n\beta/\sigma$. In particular, this implies that $P_F \equiv \alpha$ whenever $\beta = 0$, even if $\sigma < 1$. This undesirable property is not shared by the likelihood ratio test and Perng's test. For these tests, even if $\beta = 0$, the power functions approach 1 as σ approaches 0 . Thus, the F test might be expected to perform poorly relative to the other tests for alternatives where $\eta = n\beta/\sigma$

is small and σ is also small. The rather complicated form of the expressions appearing in Lemma 3.1 makes comparison among these power functions difficult other than by means of evaluation for specific choices of α, r, σ and $n\beta$.

Equations (3.6) and (3.8) have already been encountered and solving these equations presents no particular difficulty. The power functions of each of these test procedures can then be evaluated with the help of a standard numerical routine which evaluates the incomplete gamma integral provided that the integral appearing in (3.7) can be evaluated. In this connection note that

$$\sigma^{-1} \int_A^\infty \frac{\psi_{r-1}(w/\sigma)}{[\psi_{r-1}(w)]^{1/\sigma}} \, dw \leq [1 - \psi_{r-1}(A/\sigma)]/[\psi_{r-1}(A)]^{1/\sigma} .$$

For fixed r and σ, this bound is monotonically decreasing in A and consequently, for large enough A, this integral is negligible. Thus, the integral appearing in (3.7) can easily be evaluated by any of the standard routines which numerically integrate a prescribed function over a finite range.

Tables of the power functions (3.5), (3.7) and (3.9) have been evaluated for various choices of r and α for grids of $(n\beta, \sigma)$ pairs. It is rather difficult to summarize these results concisely but it does appear that the likelihood ratio test is more sensitive to departures in location than is Perng's test. More precisely, the only region of the parameter space in which the power functions of Perng's test and the likelihood ratio test seem to differ substantially is the region of σ close to 1 and $n\beta$ reasonably large where the likelihood ratio test

has a distinct advantage. In fairness, we note that Perng's test appears

to be marginally more sensitive to departures in scale. As already

noted, for fixed r and α, the power function of the F test depends

only upon $\eta = n\beta/\sigma$ and consequently behaves quite differently from

the power functions of the other two tests. It is interesting to note that the

power function of the F test tends to dominate the power functions of

both of the other tests for moderate values of $n\beta$ and the difference

can be substantial. On the other hand, as was anticipated, for the region

in which both $n\beta$ and σ are small, the F test performs poorly relative

to the other two tests. The powers for the case $\alpha = 0.05$ and $r = 10$,

20 and 40 are displayed for selected values of $n\beta$ and σ in Table 2.

Table 2 about here

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4. CONCLUDING REMARKS

We have considered three different test procedures for the particular life testing problem considered by Perng (1977). Critical values which make the likelihood ratio test easy to carry out have been provided. Comparison of the power functions of Perng's test and the likelihood ratio test reveals that although these power functions are quite similar over most of the parameter space, the likelihood ratio test has the advantage in the region of the parameter space where there is a substantial difference in these power functions. Thus, the likelihood ratio test should be preferred to Perng's test. This preference is strengthened in view of the practical difficulties associated with carrying out Perng's test. Examination of the power function of the F test reveals that in the fixed sample size case it is not possible to eliminate the F test as a viable competitor to the other two tests since there is a region of the parameter space in which the F test does substantially better than either the likelihood ratio test or Perng's test.

Table 1. Distribution of $-2 \log \lambda_n$

r	Probability of a larger value				
	0.100	0.500	0.010	0.005	0.001
2	10.678	13.490	19.958	22.735	29.175
3	8.413	10.589	15.536	17.642	22.504
4	7.631	9.606	14.076	15.971	20.335
5	7.225	9.101	13.340	15.154	19.259
6	6.973	8.789	12.851	14.626	18.612
7	6.799	8.575	12.586	14.282	18.177
8	6.670	8.417	12.384	14.033	17.863
9	6.571	8.296	12.194	13.842	17.625
10	6.492	8.200	12.059	13.691	17.437
11	6.427	8.121	11.949	13.566	17.285
12	6.373	8.055	11.858	13.466	17.156
13	6.327	7.999	11.780	13.380	17.051
14	6.287	7.950	11.713	13.305	16.960
15	6.252	7.908	11.655	13.240	16.880
16	6.221	7.871	11.604	13.183	16.810
17	6.193	7.837	11.558	13.133	16.748
18	6.168	7.808	11.517	13.088	16.693
19	6.146	7.781	11.480	13.047	16.643
20	6.126	7.756	11.447	13.010	16.598
21	6.107	7.734	11.417	12.976	16.556
22	6.090	7.713	11.389	12.945	16.519
23	6.074	7.694	11.363	12.917	16.484
24	6.060	7.677	11.339	12.890	16.452
25	6.046	7.661	11.317	12.866	16.423
30	5.990	7.594	11.227	12.766	16.302
40	5.916	7.505	11.108	12.635	16.144
50	5.868	7.448	11.031	12.550	16.042
75	5.797	7.363	10.918	12.427	15.894
100	5.757	7.315	10.855	12.357	15.811
200	5.685	7.230	10.742	12.233	15.664
500	5.624	7.158	10.648	12.131	15.543
1000	5.595	7.124	10.603	12.082	15.486
2000	5.575	7.100	10.573	12.049	15.446
5000	5.558	7.080	10.546	12.019	15.412
10000	5.549	7.069	10.532	12.005	15.395
∞	5.526	7.045	10.501	11.971	15.355

Table 2. Powers of Test Procedures

CASE: R=   10

SIGMA	TEST	N*BETA							
		0.0	1.0	2.0	2.5	3.0	3.5	4.0	4.5
1.0	LR	0.05	0.11	0.25	0.36	0.51	0.72	0.76	1.00
	PR	0.05	0.11	0.24	0.34	0.48	0.65	0.33	0.97
	F	0.05	0.14	0.34	0.48	0.62	0.75	0.84	0.91
0.9	LR	0.06	0.13	0.28	0.39	0.55	0.75	0.97	1.00
	PR	0.06	0.14	0.28	0.40	0.54	0.71	0.87	0.98
	F	0.05	0.15	0.40	0.56	0.71	0.82	0.90	0.95
0.8	LR	0.08	0.17	0.33	0.46	0.62	0.80	0.98	1.00
	PR	0.08	0.18	0.35	0.47	0.62	0.78	0.72	0.73
	F	0.05	0.17	0.45	0.66	0.50	0.89	0.95	0.93
0.7	LR	0.12	0.23	0.43	0.56	0.71	0.86	0.99	1.00
	PR	0.12	0.25	0.45	0.59	0.72	0.86	0.95	1.00
	F	0.05	0.21	0.58	0.76	0.86	0.95	0.98	0.99
0.6	LR	0.20	0.35	0.56	0.69	0.82	0.93	0.99	1.00
	PR	0.20	0.36	0.59	0.72	0.83	0.93	0.98	1.00
	F	0.05	0.26	0.71	0.87	0.95	0.93	0.99	1.00
0.5	LR	0.34	0.52	0.73	0.83	0.92	0.97	1.00	1.00
	PR	0.34	0.54	0.76	0.85	0.93	0.96	1.00	1.00
	F	0.05	0.34	0.84	0.95	0.99	1.00	1.00	1.00
0.4	LR	0.57	0.75	0.90	0.95	0.98	1.00	1.00	1.00
	PR	0.57	0.76	0.91	0.96	0.99	1.00	1.00	1.00
	F	0.05	0.48	0.95	0.99	1.00	1.00	1.00	1.00
0.2	LR	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	PR	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	F	0.05	0.95	1.00	1.00	1.00	1.00	1.00	1.00

Table 2. (cont'd)

CASE: R = 20

					N*BETA				
SIGMA	TEST	0.0	1.0	2.0	2.5	3.0	3.5	4.0	4.5
1.0	LR	0.05	0.12	0.26	0.39	0.57	0.81	1.00	1.00
	PP	0.05	0.11	0.24	0.34	0.48	0.65	0.33	0.37
	F	0.05	0.14	0.38	0.55	0.73	0.87	0.95	0.98
0.9	LR	0.07	0.15	0.32	0.46	0.64	0.85	1.00	1.00
	PR	0.08	0.16	0.32	0.44	0.58	0.74	0.89	0.99
	F	0.05	0.15	0.44	0.65	0.83	0.94	0.98	1.00
0.8	LR	0.12	0.23	0.43	0.57	0.74	0.90	1.00	1.00
	PR	0.13	0.26	0.45	0.59	0.72	0.85	0.95	1.00
	F	0.05	0.17	0.55	0.77	0.92	0.96	1.00	1.00
0.7	LR	0.22	0.38	0.60	0.73	0.86	0.96	1.00	1.00
	PR	0.25	0.42	0.63	0.75	0.85	0.94	0.98	1.00
	F	0.05	0.21	0.68	0.89	0.97	1.00	1.00	1.00
0.6	LR	0.42	0.60	0.80	0.89	0.95	0.99	1.00	1.00
	PR	0.45	0.64	0.83	0.90	0.95	0.98	1.00	1.00
	F	0.05	0.26	0.83	0.96	1.00	1.00	1.00	1.00
0.5	LR	0.70	0.84	0.95	0.98	0.99	1.00	1.00	1.00
	PR	0.73	0.87	0.96	0.98	0.99	1.00	1.00	1.00
	F	0.05	0.36	0.95	1.00	1.00	1.00	1.00	1.00
0.4	LR	0.93	0.98	1.00	1.00	1.00	1.00	1.00	1.00
	PR	0.95	0.99	1.00	1.00	1.00	1.00	1.00	1.00
	F	0.05	0.55	1.00	1.00	1.00	1.00	1.00	1.00
0.2	LR	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	PR	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	F	0.05	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 2. (cont'd)

CASE: R =   40

SIGMA	TEST	|*BETA 0.0	1.0	2.0	2.5	3.0	3.5	4.0	4.5
1.0	LR	0.05	0.12	0.23	0.41	0.61	0.86	1.00	1.00
	PR	0.05	0.11	0.24	0.34	0.48	0.65	0.83	0.97
	F	0.05	0.14	0.37	0.59	0.81	0.95	0.99	1.00
0.9	LR	0.09	0.19	0.38	0.53	0.71	0.91	1.00	1.00
	PR	0.10	0.20	0.38	0.50	0.65	0.79	0.92	0.99
	F	0.05	0.15	0.46	0.72	0.91	0.99	1.00	1.00
0.8	LR	0.20	0.35	0.58	0.71	0.85	0.95	1.00	1.00
	PR	0.24	0.36	0.61	0.72	0.83	0.92	0.98	1.00
	F	0.05	0.17	0.59	0.88	0.98	1.00	1.00	1.00
0.7	LR	0.44	0.62	0.82	0.90	0.96	0.99	1.00	1.00
	PR	0.50	0.68	0.84	0.91	0.96	0.99	1.00	1.00
	F	0.05	0.21	0.75	0.96	1.00	1.00	1.00	1.00
0.6	LR	0.77	0.89	0.97	0.99	1.00	1.00	1.00	1.00
	PR	0.81	0.92	0.97	0.99	1.00	1.00	1.00	1.00
	F	0.05	0.26	0.91	1.00	1.00	1.00	1.00	1.00
0.5	LR	0.97	0.99	1.00	1.00	1.00	1.00	1.00	1.00
	PR	0.98	0.99	1.00	1.00	1.00	1.00	1.00	1.00
	F	0.05	0.37	0.99	1.00	1.00	1.00	1.00	1.00
0.4	LR	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	PR	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	F	0.05	0.54	1.00	1.00	1.00	1.00	1.00	1.00
0.2	LR	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	PR	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	F	0.05	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Figure 1. Critical Regions of Test Procedures

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For a particular hypothesis testing problem involving the location and scale parameters of an exponential distribution, Perng (1977) proposed a test procedure based on the first r out of n observed failure times. In this paper, the likelihood ratio test for the same problem is considered. Critical values are provided and the asymptotic null distribution is determined. An alternate test based on an F statistic is also proposed and the critical regions and power functions of all three procedures are examined.